Using the limit definition of the definite integral, and right endpoints, find $\int_{-3}^{1} (x^2 + 6x + 4)\, dx$.

SCORE: _____ / 15 PTS

NOTE: Solutions using any other method will earn 0 points.

$$\Delta x = \frac{1 - -3}{n} = \frac{4}{n}$$

$$\lim_{n\to\infty} \sum_{i=1}^{n} f(a + i\Delta x)\Delta x$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} f\Big(\underbrace{-3 + \tfrac{4i}{n}}_{(1)}\Big) \underbrace{\tfrac{4}{n}}_{(1)}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \Big(\underbrace{\big(-3 + \tfrac{4i}{n}\big)^2 + 6\big(-3 + \tfrac{4i}{n}\big) + 4}_{(3)}\Big)\frac{4}{n}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(9 - \cancel{\frac{24i}{n}} + \frac{16i^2}{n^2} - 18 + \cancel{\frac{24i}{n}} + 4\right)\frac{4}{n}$$

$$= \lim_{n\to\infty} \frac{4}{n} \sum_{i=1}^{n} \Big(\underbrace{\frac{16i^2}{n^2} - 5}_{}\Big) \; (3)$$

$$= \lim_{n\to\infty} \frac{4}{n}\left(\frac{16}{n^2}\sum_{i=1}^{n} i^2 - 5n\right)$$

$$= \lim_{n\to\infty} \frac{4}{\cancel{n}}\Big(\underbrace{\frac{16}{n^2}\,\frac{\cancel{n}(n+1)(2n+1)}{6}}_{(2)} \underbrace{- 5\cancel{n}}_{(2)}\Big)$$

$$= \underbrace{4\left(\frac{16 \cdot \cancel{2}}{\cancel{6}\,3} - 5\right)}_{(1)}$$

$$= \underbrace{\frac{4}{3}}_{(1)}$$

(1) IF YOU WROTE $\lim\limits_{n\to\infty} \sum\limits_{i=1}^{n}$ ON EVERY LINE BEFORE THIS

A person's velocity (in meters per ~~minute~~ SECOND) at time t (in ~~minutes~~ SECONDS) is given by $v(t) = \begin{cases} 13-2t, & 0 \le t \le 5 \\ t-2, & 5 \le t \le 12 \end{cases}$.

SCORE: _____ / 5 PTS

[a] Estimate the distance the person travelled from time $t = 0$ seconds to $t = 12$ seconds using three subintervals and right endpoints.

$\Delta t = \frac{12-0}{3} = 4$

$v(4)\Delta t + v(8)\Delta t + v(12)\Delta t = \underbrace{5(4)}_{} + \underbrace{6(4)}_{} + \underbrace{10(4)}_{} = \underbrace{84}_{}$ METERS

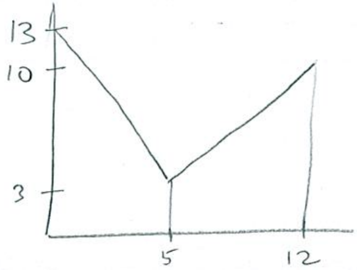
$\frac{1}{2}$ $\frac{1}{2}$ 1 $\frac{1}{2}$

[b] Find the exact distance the person travelled from time $t = 0$ seconds to $t = 12$ seconds.

$\frac{1}{2}(13+3)(5-0) + \frac{1}{2}(3+10)(12-5)$

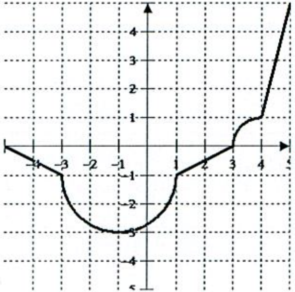
$= \underbrace{\frac{1}{2}(13+3)(5)}_{} + \underbrace{\frac{1}{2}(3+10)(7)}_{}$ 1 1

$= 40 + \frac{91}{2} = \underbrace{\frac{171}{2} \text{ OR } 85\frac{1}{2}}_{}$ METERS $\frac{1}{2}$



The graph of function f is shown on the right. **SCORE: _____ / 5 PTS**

The graph consists of 3 diagonal lines, alternating with two arcs of circles.



$\frac{1}{2}$ POINT EACH

[a] Evaluate $\int_{-5}^{5} f(x)\,dx$.

$$= \int_{-5}^{3} f(x)\,dx + \int_{3}^{5} f(x)\,dx$$

$$= -\left[\tfrac{1}{2}(1)(2) + \left(\tfrac{1}{2}\pi(2)^2 + 1(4)\right) + \tfrac{1}{2}(1)(2)\right] + \tfrac{1}{4}\pi(1)^2 + \tfrac{1}{2}(1+5)(1)$$

$$= -[1 + 2\pi + 4 + 1] + \tfrac{1}{4}\pi + 3 = -\tfrac{7}{4}\pi - 3$$

[b] Evaluate $\int_{4}^{1} f(x)\,dx$.

$$= -\int_{1}^{4} f(x)\,dx = -\left[\int_{1}^{3} f(x)\,dx + \int_{3}^{4} f(x)\,dx\right] = -\left[-1 + \tfrac{\pi}{4}\right] = 1 - \tfrac{\pi}{4}$$

Evaluate $\int_0^3 (4\sqrt{9-x^2} - |x-1|)\,dx$ using the properties of definite integrals and interpreting in terms of area. **SCORE: _____ / 5 PTS**

$$\underbrace{= 4\int_0^3 \sqrt{9-x^2}\,dx - \int_0^3 |x-1|\,dx}_{2} = 4\left(\tfrac{1}{4}\pi(3)^2\right) - \left[\tfrac{1}{2}(1)(1) + \tfrac{1}{2}(2)(2)\right]$$

$$= \underbrace{9\pi}_{1\frac{1}{2}} - \underbrace{\frac{5}{2}}_{1\frac{1}{2}}$$